Preliminary Control System Using Preferred Feedback States

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Nomenclature

M_δ, M_α = vehicle body angular acceleration gains of fin deflection δ and angle of attack α, (deg/sec^2)/deg
Z_α = vehicle lateral acceleration gain of α, g/deg
K_v = fin servo valve gain, (deg/sec)/deg
K_δ, K_R, K_z = feedback gains

Theme

THE objectives of this paper are twofold: first, to present the fin position δ and body rate $\dot{\theta}$ feedback shaping transfer functions that equate a conventional fin feedback autopilot to a reference autopilot that replaces fin feedback with body angular acceleration $\ddot{\theta}$ as a feedback signal. The word "reference" is used because there are no practical angular accelerometer sensors available for measuring body angular accelerations, and also, for analysis reasons, the angular acceleration loop is assumed to be free of the tolerance problem that is inherent in matching M_δ in the feedback path with M_δ in the forward path. The second objective is to demonstrate the practical application of this control scheme by analysis and examples with the end view being to facilitate rigid-body autopilot design work and to maximize the controllable aerodynamic regime of vehicles that are subject to large M_α.

Contents

This paper presents the fin and rate feedback shaping transfer functions that equate a conventional fin feedback autopilot to a reference autopilot employing body angular acceleration as a feedback signal. That shaping is referred to as the "angular acceleration model" and is presented in Fig. 1 in a simplified lateral acceleration control autopilot. The rate feedback shaping transfer function TF_1 is used if the airframe is aerodynamically unstable; the sign of M_α is positive. The rate feedback shaping transfer TF_2 is used if the airframe is aerodynamically stable; the sign of M_α is negative. Figure 2 presents the equivalent lateral acceleration control autopilot using body angular acceleration feedback instead of fin position feedback. The two are equivalent in that the transfer function Z/Z_c shown by Eq. (1) is equal for both autopilots:

$$\frac{Z}{Z_c}=\frac{K_v M_\delta Z_\alpha K_z}{S^3+K_v K_\delta S^2+(K_v M_\delta K_R-M_\alpha)S+K_v M_\delta Z_\alpha K_z}\frac{g's}{g} \tag{1}$$

In essence, the model shaping blocks fin feedback and differentiates rate feedback below the square root of M_α break frequency.

The transfer function Z/Z_c of the fin feedback autopilot without any shaping is shown by

$$\frac{Z}{Z_c}=\frac{K_v M_\delta Z_\alpha K_z}{S^3+K_v K_\delta S^2+(K_v M_\delta K_R-M_\alpha)S+K_v(M_\delta Z_\alpha K_z-K_\delta M_\alpha)} \tag{2}$$

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The appearance of M_α in the constant term of the characteristic denominator complicates the static as well as the dynamic control system synthesis problem. By comparison, the synthesis problem is simplified by working directly from the reference autopilot to determine the feedback gains, and then converting to the equivalent fin feedback or "modeled" autopilot by means of the angular acceleration model. Examples given subsequently illustrate a systematic way of obtaining feedback gains from the reference autopilot to meet control system response time and stability margin requirements of a fifth-order plant.

Reference Autopilot Analysis

Equation (3) is the stability equation found by applying the Routh-Hurwitz criteria to the charactertistic denominator of the reference autopilot:

$$K_v M_\delta K_R \geq \frac{M_\delta Z_\alpha K_z}{K_\delta}+M_\alpha \tag{3}$$

Parameter margins are given by

$$K_v=m_1\left(\frac{Z_\alpha K_z}{K_\delta}+\frac{M_\alpha}{M_\delta}\right)\frac{1}{K_R} \tag{4}$$

$$M_\delta=m_2\left(\frac{K_\delta M_\alpha}{K_v K_\delta K_R-Z_\alpha K_z}\right) \tag{5}$$

$$Z_\alpha=\frac{1}{m_3}(K_v M_\delta K_R-M_\alpha)\frac{K_\delta}{M_\delta K_z} \tag{6}$$

where m_1, m_2, and m_3 are the gain margins on K_v, M_δ, and Z_α, respectively. The gain margins on K_v and Z_α will be the same in both the reference and modeled autopilots. The gain margin on M_δ in the modeled autopilot was found to be approximately equal to the square root of the gain margin on M_δ in the reference autopilot.

Autopilot Parameter Values and Margin/Response Requirements

The following aerodynamic parameter values and stability margins were used in the analysis: aerodynamic gains – $Z_\alpha=1.0$, $M_\delta=100$, M_α to be determined from the examples; valve-actuator – $K_v=100$, $F(S)=35500/[S^2+(2)(0.25)(188.4)S+35500]$; response requirements – transient rise time to a step command Z_c to be around $\tau=⅓$ sec; steady-state accuracy = 1.0; margins requirements – minimum gain and phase margins are $K_v \pm 9$dB, $M_\delta \pm 6$dB, $\phi \geq 30°$. $K_\delta=0.314$ gives a factor-of-3 margin against valve gain K_v going up and driving the autopilot unstable. K_δ is determined from either the fin loop or the body angular acceleration loop, which are approximately equal to $(K_v/S)F(S)K_\delta$ at high frequency, and is essentially the total of the three-loop Nyquist gain at high-frequency crossover. Thus, $(K_v/S)F(S)K_\delta=-0.333$ at the w_n frequency of 188.4 rad/sec. To simplify the preliminary analysis $F(S)=1$ is used.

Example 1: Airframe Unstable

This example presents a systematic way of obtaining control system feedback gains from the Routh stability equations. The feedback gains then are used to convert the reference autopilot into the modeled autopilot.

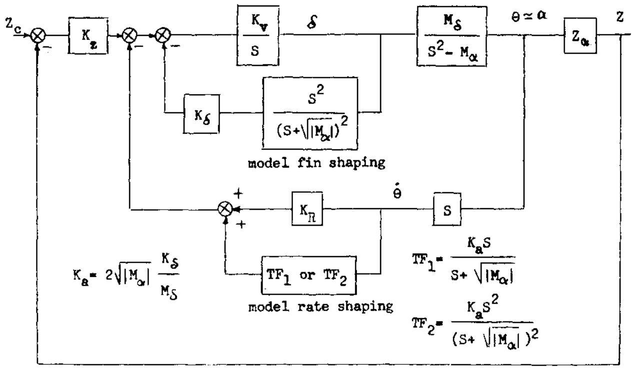

Fig. 1 Lateral acceleration control autopilot using the angular acceleration model.

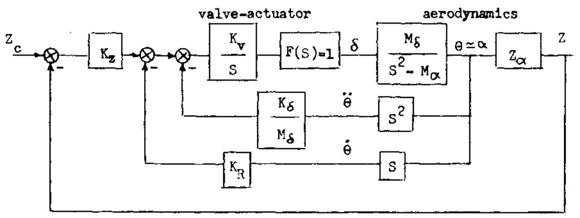

Fig. 2 Lateral acceleration control reference autopilot using angular acceleration feedback.

From the Routh stability Eq. (4) a value of $m_l = 3.0$ gives a factor-of-3 margin against valve gain K_v going down and causing instability. From Eq. (5) a value of $m_2 = 4.0$ is needed, because of the square root relationship stated earlier, if there is to be 6 dB of margin on the aero gain M_δ decreasing and causing instability in the modeled autopilot. These two stability equations can be put in the form of

$$K_v M_\delta K_R = m_l\left(\frac{M_\delta Z_\alpha K_z}{K_\delta} + M_\alpha\right) = 954K_z + 3M_\alpha \tag{7}$$

$$K_v M_\delta K_R = \frac{M_\delta Z_\alpha K_z}{K_\delta} + m_2 M_\alpha = 318K_z + 4M_\alpha \tag{8}$$

The more stringent of the two equations is Eq. (8); this fact will be verified later. The lateral acceleration feedback gain K_z is computed in terms of M_α for a system root at $S = -3.0$ using the denominator of Eq. (1) with Eq. (8) substituted into the coefficient of the S term. The results are shown by

$$\begin{array}{r} S^2 + 28.4S + 318K_z + 3M_\alpha - 85.2 \\ \hline S+3\,\big\lceil\, S^3 + 31.4S^2 + (318K_z + 4M_\alpha - M_\alpha)S + 10^4 K_z \end{array} \tag{9}$$

where

$$10^4 K_z - (954K_z + 9M_\alpha - 256) = 0.0$$

is the remainder of Eq. (9) set equal to zero. K_z is computed from the remainder of Eq. (9), resulting in

$$K_z = 9.95(10^{-4})M_\alpha - 0.0283 \tag{10}$$

The divisor and quotient of Eq. (9) are the roots of the reference autopilot, but, since it is only third order, any large value of M_α can be controlled. A conservative maximum for M_α is determined if the damping ratio of the quadratic is made $\zeta \geq = 0.9$. M_α then can be computed using

$$\begin{aligned} w_n^2 &= 318K_z + 3M_\alpha - 85.2 \\ &= 318(0.000995M_\alpha - 0.0283) + 3M_\alpha - 85.2 \\ &= 3.32M_\alpha - 94.2 \end{aligned} \tag{11}$$

Let

$$\zeta = 0.9 = 28.4/2w_n \qquad w_n^2 = 249 \tag{12}$$

$$3.32M_\alpha - 94.2 = 249 \qquad M_\alpha \simeq 100 \tag{13}$$

The feedback gains K_z and K_R are computed from Eqs. (10) and (8), and a check on K_v margin is made with Eq. (7). The margin on Z_α is found from Eq. (6). Equations (14-17) show those computations.

$$K_z = 0.000995(100) - 0.0283 \qquad K_z = .0712 \tag{14}$$

$$10^4 K_R = 318(0.0712) + 400 \qquad K_R = 0.0423 \tag{15}$$

$$K_v M_\delta K_R \geq 954K_z + 3M_\alpha$$

$$10^4(0.0423) \geq 954(0.0712) + 300 \quad 423 \geq 366 \tag{16}$$

$$m_3 = (423 - 100)(0.314/7.12) \quad m_3 = 14.2 \tag{17}$$

The roots of the reference autopilot that has been designed are shown by

$$\frac{Z}{Z_c} = \frac{712}{(S+3)(S+14.2 \pm j6.0)} \tag{18}$$

The roots of the modeled autopilot are shown by

$$\frac{Z}{Z_c} = \frac{712(S+10)^2}{(S+3)(S+14.2 \pm j6.0)(S+10)^2} \tag{19}$$

Nyquist plots that include $F(S)$ now can be constructed to check phase and gain margins and to ascertain if a larger M_α could be modeled. If not, the effects on control system margins and response at $M_\alpha = 0$ can be investigated assuming the square root of $|M_\alpha|$ to be a constant whose value is 10.

Example 2: Airframe Stable

The appropriate Routh stability equation is Eq. (6). The stability equation is put in the form of

$$K_v M_\delta K_R = m_3 \frac{M_\delta Z_\alpha K_z}{K_\delta} + M_\alpha \tag{20}$$

and the roots and feedback gains of the reference autopilot then are calculated as in Example 1, but expressed in general terms. (The opening statements of Example 2 supply the reader with the correct starting place if he wants to iterate the preceding steps for the stable airframe condition.)